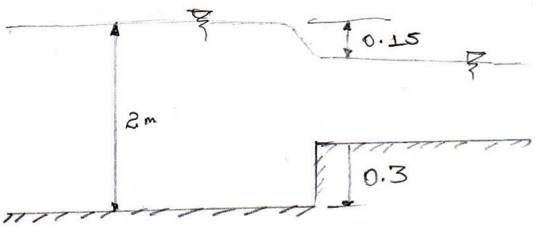
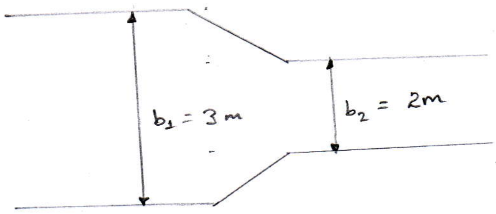
15.17

For purpose of discharge measurement the width of a rectangular channel is reduced gradually from 3m to 2m and the floor is raised by 0.3 m at a given section. What rate of flow will be indicated if the approaching depth of flow is 2m, and the wate surface elevation at the contracted section drops by 0.15 m.




- From Bernoulli's principle.

$$E_1 = E_2 + \Delta Z.$$

$A_1 = by.$
$= 3 \times 2 = 6m^2$
$A_2 = 2 \times 1.55 = 3.1m^2$

$$\Rightarrow \quad y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta Z.$$

$$\Rightarrow \quad y_1 + \frac{Q^2}{2gA_1^2} = y_2 + \frac{Q^2}{2gA_2^2} + \Delta Z.$$

$$\Rightarrow \quad 2 + \frac{Q^2}{2(9.81)(6^2)} = 1.55 + \frac{Q^2}{2(9.81)(3.1)^2} + 0.3$$

$$\Rightarrow \quad 0.0053Q^2 - 0.0014Q^2 = 2 - 1.85$$

$$\Rightarrow \quad 0.0039Q^2 = 0.15$$

$$\Rightarrow \quad Q^2 = 38.46.$$

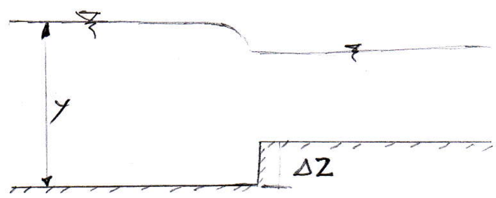
$$\Rightarrow \quad Q = 6.21 \ m^3/s$$

15.16

In a rectangular channel 3.5m wide laid at a slope of 0.0036, uniform flow occurs at a depth of flow of 2m. Assuming manning's $n = 0.015$.

1) Calculate the height of a hump in the flow could be raised without causing afflux?

2) If the upstream depth of flow is to be raised to 2.5m, what would be the height of the hump?




Given

$b = 3.5\,m$

$S = 0.0036$

$y = 2m$

$n = 0.015$

$A = by$
$= 3.5 \times 2 = 7m^2$

$P = b + 2y$
$= 3.5 + 2(2) = 7.5m$

$R = \frac{A}{P} = \frac{7}{7.5} = 0.93\,m$

a) $q = \frac{Q}{b} = \frac{AV}{b} = \frac{\cancel{b}yV}{\cancel{b}} = yV$

$= 2\left(\frac{1}{n} R^{2/3} S^{\frac{1}{2}}\right)$

$= 2\left(\frac{1}{0.015}(0.93)^{2/3}(0.0036)^{\frac{1}{2}}\right)$

$= 7.62\,m^2/s$

$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$

$= \left(\frac{7.62^2}{9.81}\right)^{\frac{1}{3}} = 1.81m$

$E_c = \frac{3}{2}(y_c) = \frac{3}{2}(1.81) = 2.715$

Energy in the upper stream. $E_1$.

$E_1 = y_1 + \frac{V^2}{2g}$

$= 2 + \frac{q^2}{2gy^2}$

$= 2 + \frac{7.62^2}{2(9.81)2^2}$

$= 2.74m.$

Since $E_1 > E_c$ there is no afflux.

$\Delta Z_{max} = E_1 - E_c$

$= 2.74 - 2.715$

$= 0.025m$ (Ans)

b)

b) $y = 2.5m.$

$\Delta Z = ?$

$E = y + \frac{q^2}{2gy^2}.$

$= 2.5 + \frac{7.62^2}{2 \times 9.81 \times 2.5^2}$

$= 2.974\,m$

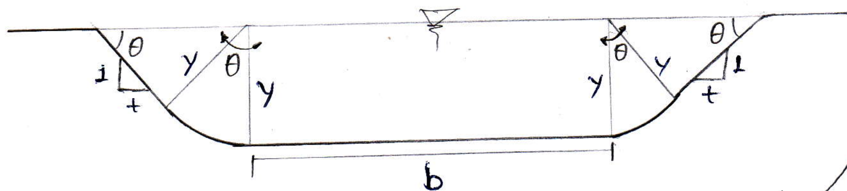
$\Delta z = E - E_c.$

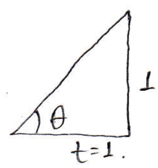
$= 2.974 - 2.715$

$= 0.259\,m$ (Ans)

15.21

Design a concrete lined channel of the form shown in the figure. The channel has to carry a discharge of $500\,m^3/s$ at a slope of 1 in 4000. The side slope of a channel may be taken as 1H:1V. The permissible velocity of flow in the section is 2.5 m/s. For the lining, assume Mannings roughness coefficient, $n = 0.014$.




$\theta = \tan^{-1}(1)$

$\theta = \frac{\pi}{4} = 45°$

**Given**

$Q = 500\,m^3/s$

$V = 2.5\,m/s$

$S = \frac{1}{4000}$

$t = 1$

$n = 0.014$

$\theta = \frac{\pi}{4}$

$A = y(b + y\cot\theta + y\theta)$

$P = (b + 2y\cot\theta + 2y\theta)$

$\therefore A = y(b + y + y(\frac{\pi}{4}))$

$P = b + 2y + y(\frac{\pi}{2})$

$$Q = AV$$

$\Rightarrow A = \frac{Q}{A}$

$\Rightarrow A = \frac{500}{2.5}$

$\Rightarrow A = 200\,m^2$

$V = \frac{1}{n} R^{2/3} S^{\frac{1}{2}}$

$\Rightarrow V = \frac{1}{n}\left(\frac{A}{P}\right)^{2/3} S^{\frac{1}{2}}$

$\Rightarrow V = \frac{S^{\frac{1}{2}}}{n} \cdot \frac{A^{2/3}}{P^{2/3}}$

$\Rightarrow P^{2/3} = \frac{S^{\frac{1}{2}}}{n} \cdot \frac{A^{2/3}}{V}$

$\Rightarrow P = \left(\frac{S^{\frac{1}{2}}}{n} \cdot \frac{A^{2/3}}{V}\right)^{3/2}$

$\Rightarrow P = \left(\frac{(0.00025)^{\frac{1}{2}}}{0.014} \cdot \frac{(200)^{2/3}}{2.5}\right)^{3/2}$

$\Rightarrow P = 60.73\,m$

$\therefore 200 = y(b + y + y(\frac{\pi}{4}))$ .... (1)

$60.73 = b + 2y + y(\frac{\pi}{2})$ ... (2)

from equation (2) b can be written as.

$b = 60.73 - 2y - y(\frac{\pi}{2})$ ... (3)

Substituting equation (3) into (1) gives.

$\Rightarrow 200 = y(60.73 - 2y - y\frac{\pi}{2} + y + y(\frac{\pi}{4}))$

$\Rightarrow 200 = y(60.73 - 1.785y)$

$\Rightarrow 200 = 60.73y - 1.785y^2$

$\Rightarrow 0 = -1.785y^2 + 60.73y - 200$

$\Rightarrow y = \frac{-60.73 \pm \sqrt{60.73^2 - 4(-1.785)(-200)}}{2(-1.785)}$

$\Rightarrow y = 3.694$ and $y = 30.33$

$\therefore b = 60.73 - 2(3.694) - 3.694(\frac{\pi}{2})$ when $y = 3.694$

$b = 47.54$

$b = 60.73 - 2(30.33) - 30.33(\frac{\pi}{2})$ when $y = 30.33$

$b = -47.57$

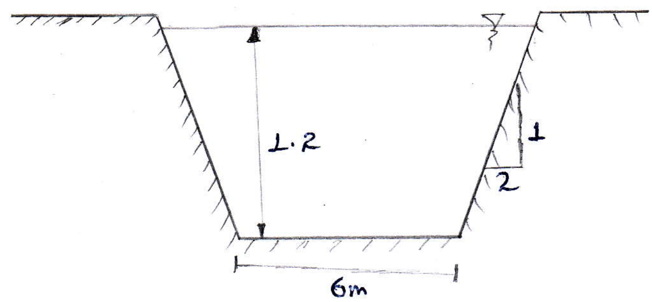
$\therefore$ The dimensions of the channel are.

$y = 3.694\,m$ & $b = 47.54\,m$ (Ans)

15.15

A trapezoidal channel has a bottom width of 6m and side slopes of 2 horizontal to 1 vertical. If the depth of flow is 1.2m at a discharge of $10 m^3/s$, Compute.

1. The specific energy
2. The critical depth.




Given

$b = 6m$
$t = 2$
$y = 1.2m$
$Q = 10 m^3/s$

1) The specific Energy.

$E = E_s + E_k$ $\qquad E_s = y$

$E = y + \alpha \frac{V^2}{2g}$ $\qquad E_k = \alpha \frac{V^2}{2g}$

* $\alpha = 1$, for uniform flow.

$$E = 1.2 + \frac{(0.992)^2}{2 \times 9.81}$$
$$= 1.2 + 0.05$$
$$= 1.25 m$$

$Q = AV$

$\Rightarrow V = \frac{Q}{A}$ $\qquad A = by + ty^2$
$\qquad = 6(1.2) + 2(1.2)^2$
$\qquad = 10.08 m^2$

$\Rightarrow V = \frac{10}{10.08}$

$\Rightarrow V = 0.992 m/s$

2) The critical depth

- is the critical flow condition where E is minimum.

$$\frac{dE}{dy} = \frac{d}{dy}\left(y + \frac{Q^2}{2gA^2}\right) = 0$$
$$= 1 + \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right) = 0$$
$$= 1 - \frac{Q^2}{gA^3}\frac{dA}{dy} = 0$$

$$\frac{dA}{dy} = \frac{d(by + ty^2)}{dy}$$
$$= b + 2ty$$

$$\therefore \boxed{\frac{Q^2}{gA^3}(b + 2ty) = 1}$$

$$\Rightarrow \boxed{\frac{Q^2}{g} = \frac{(by + ty^2)^3}{(b + 2ty)}}$$

$$\frac{10^2}{9.81} = \frac{(6y + 2y^2)^3}{(6 + 2(2)y)}$$

$$10.194 = \frac{(6y + 2y^2)^3}{(6 + 4y)}$$

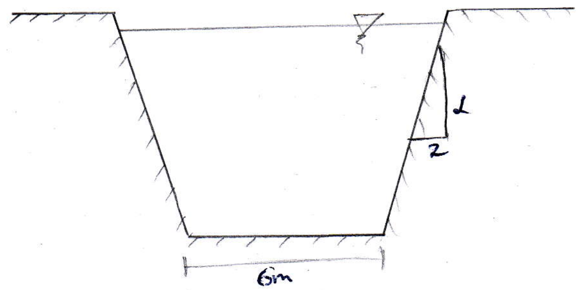
by trial and error method.

$$y_c = 0.6113 m$$

15.9

A trapezoidal channel having bottom width 6m and side slope of 2H:1V is laid on a bottom slope of 0.0016. It carries a uniform flow of water at the rate of $10\ m^3/s$. Assuming Manning's $n = 0.025$, calculate

1. The normal depth of flow
2. The mean velocity of flow.




<u>Given</u>

$b = 6m$
$t = 2$
$S = 0.0016$
$Q = 10\ m^3/s$
$n = 0.025.$

$$A = by + ty^2$$
$$= 6y + 2y^2$$

$$P = 2y\sqrt{1+t^2} + b$$
$$= 2y\sqrt{1+2^2} + 6.$$
$$= 2\sqrt{5}\,y + 6$$

$$Q = \frac{1}{n} \cdot \frac{A^{5/3}}{P^{2/3}} \cdot S^{\frac{1}{2}}$$

$$\Rightarrow \frac{Qn}{S^{\frac{1}{2}}} = \frac{A^{5/3}}{P^{2/3}}$$

$$\Rightarrow \frac{10 \times 0.025}{(0.0016)^{\frac{1}{2}}} = \frac{(6y + 2y^2)^{5/3}}{(2\sqrt{5}\,y + 6)^{2/3}}$$

$$\Rightarrow 0 = \frac{(6y + 2y^2)^{5/3}}{(2\sqrt{5}\,y + 6)^{2/3}} - 6.25$$

Numerical method (Bisection)

| a | c | b | f(c) |
|---|---|---|---|
| 0.75 | 0.875 | 1 | -0.98527 |
| 0.875 | 0.9375 | 1 | -0.29550 |
| 0.9375 | 0.96875 | 1 | +0.06479 |
| 0.9375 | 0.953125 | 0.96875 | -0.11664 |
| 0.953125 | 0.9609375 | 0.96875 | -0.02625 |
| 0.9609375 | 0.96484375 | 0.96875 | +0.0191896 |
| 0.9609375 | 0.962890625 | 0.96484375 | - |
| 0.962890625 | 0.963867 | 0.96484375 | +0.007820 |

$\therefore y_n = 0.964\ m$

* The mean velocity

$$V = \frac{1}{n} \cdot R^{2/3} \cdot S^{\frac{1}{2}}$$

$$= \frac{1}{0.025} \cdot (0.741)^{2/3} \cdot (0.0016)^{\frac{1}{2}}$$

$$= 1.31\ m/s$$

$$A = 6y + 2y^2$$
$$= 6(0.964) + 2(0.964)^2$$
$$= 7.642\ m^2$$

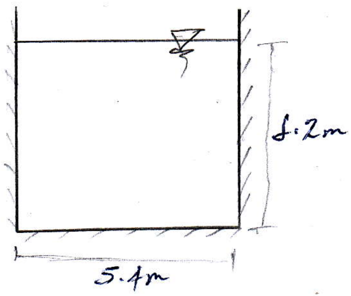
$$P = 2\sqrt{5}(0.964) + 6$$
$$= 10.31\ m$$

$$R = \frac{A}{P}$$
$$= \frac{7.642}{10.31}$$
$$= 0.741$$

15.8

A rectangular channel 5.4 m wide and 1.2 m deep has a slope of 1:1000 and is lined with good rubble masonary for which Manning's $n = 0.017$. It is desired to increase the discharge to a maximum by changing the channel slope or the form of the section. The dimension of the section may be changed but the channel must contain the same amount of lining.

Compute the new dimensions and probable increase in discharge.




Given

$S = 0.001$

$n = 0.017$

$$A = 5.4 * 1.2 = \underline{6.48\ m^2}$$

$$P = 2(1.2) + 5.4 = \underline{7.8}$$

$$R = \frac{A}{P} = \frac{6.48}{7.8} = \underline{0.831\ m}$$

$$Q = A.V = \frac{1}{n} \cdot A \cdot R^{2/3} \cdot S^{\frac{1}{2}}$$

$$= \frac{1}{0.017} * 6.48 * 0.831^{2/3} * 0.001^{\frac{1}{2}}$$

$$= \underline{10.654\ m^3/s}$$

* Inorder to calculate the maximum discharge we have to differentiate Q with rispect to b and y.

$$A = by$$
$$P = b + 2y.$$

$$Q = A.V = \frac{1}{n} \cdot \frac{A^{5/3}}{P^{2/3}} \cdot S^{\frac{1}{2}}$$

$$= \frac{S^{\frac{1}{2}}}{n} \cdot \frac{(by)^{5/3}}{(b+2y)^{2/3}}$$

$$\frac{\partial Q}{\partial b} = 0, \quad \frac{\partial \left( \frac{(by)^{5/3}}{(b+2y)^{2/3}} \right)}{\partial b} = 0$$

$$\Rightarrow \frac{\frac{5}{3}(by)^{2/3} \cdot y (b+2y)^{2/3} - (by)^{5/3} \cdot \frac{2}{3}(b+2y)^{-\frac{1}{3}}}{\left( (b+2y)^{2/3} \right)^2} = 0$$

$$\Rightarrow \frac{5}{3} y (by)^{2/3} (b+2y)^{2/3} - \frac{2}{3}(by)^{5/3} (b+2y)^{-\frac{1}{3}} = 0 \quad \text{---} \quad (1)$$

$$\frac{\partial Q}{\partial y} = \frac{\frac{5}{3}(by)^{2/3} b (b+2y)^{2/3} - (by)^{5/3} \cdot \frac{2}{3} (b+2y)^{-\frac{1}{3}} \cdot 2}{\left( (b+2y)^{2/3} \right)^2} = 0$$

$$\frac{5b}{3}(by)^{2/3} (b+2y)^{2/3} - \frac{4}{3}(by)^{5/3} (b+2y)^{-\frac{1}{3}} = 0 \quad \text{---} \quad (2)$$

Solving equation 1 and 2 symultaniously gives.

$$\frac{5}{3}y\,\cancel{(by)}^{2/3}\,(b+2y)^{2/3} - \frac{2}{3}\cancel{(by)}^{5/3}\,\cancel{(b+2y)}^{-1/3} = 0$$

$$\frac{5}{3}b\,\cancel{(by)}^{2/3}\,\cancel{(b+2y)}^{2/3} - \frac{4}{3}\cancel{(by)}^{5/3}\,\cancel{(b+2y)}^{-1/3} = 0$$

$$-2\begin{cases}\frac{5}{3}y - \frac{2}{3} = 0\\ \frac{5}{3}b - \frac{4}{3} = 0\end{cases}$$

$$\frac{5b}{3} - \frac{10y}{3} - \frac{4}{3} + \frac{4}{3} = 0$$

$$\frac{5b}{3} = \frac{10y}{3}$$

$$\underline{b = 2y}$$

* The information given above says the channel must contain the same amount of lining. which means that the total surface are that is in contact with the water is the same (equal)

$P'_{(previous)}L = P_{new}L$ $\qquad$ $P_{previous}$ = previous wetted perimeter

$P_{previous} = P_{new}$. $\qquad$ $P_{new}$ = new wetted perimeter

$L$ = length of the channel.

$$\underline{P_{new} = 7.8\,m}$$

$P = b + 2y$

$\Rightarrow P = 2y + 2y$

$\Rightarrow P = 4y$

$\Rightarrow y = \frac{P}{4}$

$y = \frac{7.8}{4}$

$= \underline{1.95\,m}$

$b = 2y$

$b = 2(1.95)$

$\underline{b = 3.9\,m}$

$A = by$

$= 3.9 \times 1.95$

$= \underline{7.605\,m^2}$

$R = \frac{A}{P}$.

$R = \frac{7.605}{7.8}$

$\underline{R = 0.975}\,m$

∴ The maximum discharge is

$$Q = \frac{1}{n}\cdot A\cdot R^{2/3}\cdot S^{\frac{1}{2}}$$

$$= \frac{1}{0.017} \times 7.605 \times 0.975^{2/3} \times 0.001^{\frac{1}{2}}$$

$$= \underline{13.91\,m^3/s}$$

⇒ Increase in discharge.

$$= \frac{Q_{new} - Q_{old}}{Q_{new}} \times 100\%$$

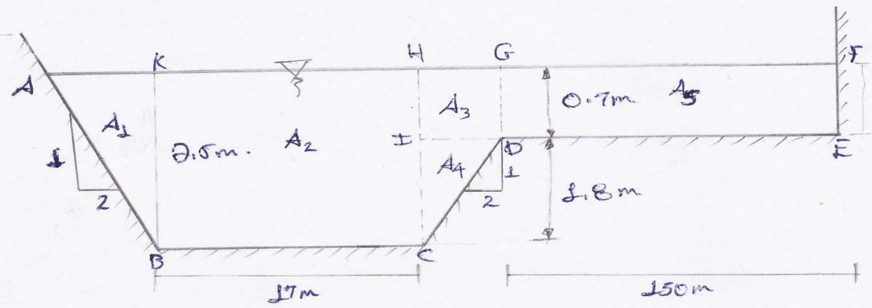
$$= \frac{13.91 - 10.654}{13.91} \times 100\%$$

$$= \underline{23.41\,\%}$$

1

15.7.

An earth canal section in good condition is 17 m wide at the bottom and has side slope of 2H:1V. One side slope extends to a height of 7.8 m above the bottom level and the other side extends to an elevation of 1.8 m, then extends flat to a distance of 150 m and rises vertically as shown in the figure. The bed slope of the canal is 0.7/1610 m/m. Assuming Chezy's C = 35, estimate the discharge when the depth of flow is 2.5 m.




Given

$S = \frac{0.7}{1610} = 0.000435$

$C = 35$

$Q = A.V$

$= A.C\sqrt{RS}$

$Q = 159.51 \times 35 \times \sqrt{0.9 \times 0.000435}$

$= 110.44\ m^3/s$

$A = A_1 + A_2 + A_3 + A_4 + A_5$

$A_1 = \frac{1}{2} \times 2(2.5) \times 2.5 = 6.25\ m^2$

$A_2 = 17 \times 2.5 = 42.5\ m^2$

$A_3 = 0.7 \times 2(1.8) = 2.52\ m^2$

$A_4 = \frac{1}{2} \times 1.8 \times 2 \times 1.8 = 3.24\ m^2$

$A_5 = 150 \times 0.7 = 105\ m^2$

$A = 6.25 + 42.5 + 2.52 + 3.24 + 105$

$= 159.51\ m^2$

$P = AB + BC + CD + DE + EF$

$= 2.5\sqrt{5} + 17 + 1.8\sqrt{5} + 150 + 0.7$

$= 177.315\ m$

$R = \frac{A}{P}$

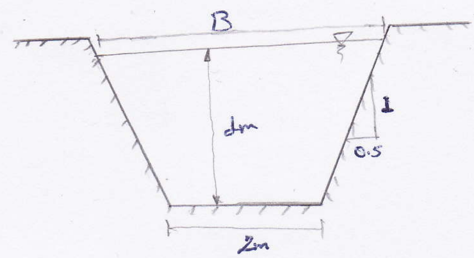
$= \frac{159.51}{177.315}$

$= 0.9\ m$

15.4.

An earthen channel with a base width of 2m and side slope of 1H: 2V carries water with a depth of 1m. The bed slope is 1 in 625. Assume Manning's roughness coefficient $n = 0.03$ and calculate.

1. Discharge through the channel.
2. The average shear stress at the channel boundary.




Give

$b = 2m$

$t = 0.5$

$y = 1m$

$S = \frac{1}{625}$

$n = 0.03$

$w = 9810\ N/m^3$

$\sin\theta \cong S$

$$Q = A.V$$
$$= A.\frac{1}{n}.R^{2/3}.S^{\frac{1}{2}}$$
$$= \frac{1}{0.03} * 2.5 * (0.59)^{2/3} * \frac{1}{625^{\frac{1}{2}}}$$
$$= 2.345\ m^3/s$$

$$A = by + ty^2$$
$$= 2(1) + 0.5(1)^2$$
$$= 2 + 0.5$$
$$= 2.5\ m^2$$

$$P = 2y\sqrt{1+t^2} + b$$
$$= 2(1)\sqrt{1+0.5^2} + 2m$$
$$= 4.236\ m$$

$$R = \frac{A}{P}$$
$$= \frac{2.5}{4.236}$$
$$= 0.59\ m$$

$$\tau = wR\sin\theta$$
$$= 9810 * 0.59 * 0.0016$$
$$= 9.26\ N/m^2$$

Ans: $Q = 2.345\ m^3/s$

$\tau = 9.26\ N/m^2$

15.3

A flow of 100 L/s flows down in a rectangular laboratory flume of width 0.6 m and having adjustable bottom slope. If the Chezy's C is 56, determine

1. The bed slope necessary to maintain a uniform flow with a depth of flow of 0.3 m.
2. The Conveyance
3. The state of flow. (Tranquil or Rapid)

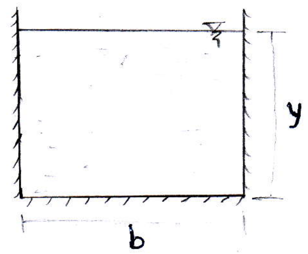
Given

$Q = 100 \, L/s = 0.1 \, m^3/s$

$b = 0.6 \, m$

$C = 56$

$S = ?$ if $y = 0.3 \, m$




Sol$^n$

1) The bed slope.

$$Q = AV$$
$$= A \cdot C\sqrt{RS}$$
$$S^{\frac{1}{2}} = \frac{Q}{ACR^{\frac{1}{2}}}$$

$$A = by$$
$$= 0.6 * 0.3$$
$$= 0.18 \, m^2$$

$$P = b + 2y$$
$$= 0.6 + 2(0.3)$$
$$= 1.2 \, m$$

$$R = \frac{A}{P}$$
$$R = \frac{0.18}{1.2}$$
$$R = 0.15 \, m$$

$$S^{\frac{1}{2}} = \frac{0.1}{0.18 * 56 * (0.15)^{\frac{1}{2}}}$$
$$S^{\frac{1}{2}} = 0.0256$$
$$S = 0.000656$$
$$\Rightarrow S = 1 \text{ in } 1524.4$$

2) The Conveyance.

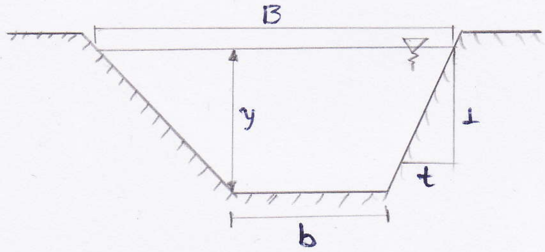
$$K = \frac{Q}{S^{\frac{1}{2}}}$$
$$K = \frac{0.1}{(0.000656)^{\frac{1}{2}}}$$
$$K = 3.904 \, m^3/s$$

3) State of flow

15.1,

An irrigation channel of trapezoidal section, having side slopes of 3H:1V is to carry a flow of 10 $m^3/s$ on a longitudinal slope of 1 in 5000. The channel is to be lined for which the Manning's roughness Coefficient is estimated as $n = 0.012$.

Find the dimensions of the most economic section of the Channel.




Given

$t = 3$

$Q = 10\ m^3/s$

$S = 0.0002$

$n = 0.012$

$$Q = AV$$
$$= A \cdot \frac{1}{n} R^{2/3} S^{1/2}$$
$$= \frac{1}{n} \cdot \frac{A^{5/3}}{P^{2/3}} S^{1/2}$$

* The most economical dimensions for trapezoidal Channel section are.

$$\frac{b+2ty}{2} = y\sqrt{1+t^2} \qquad b = 2y\sqrt{1+t^2} - 2ty$$
$$b = 2y\left(\sqrt{1+t^2} - t\right)$$
$$R = \frac{y}{2}$$

$$\therefore Q = A \cdot V \qquad A = by + ty^2$$
$$= A \cdot \frac{1}{n} R^{2/3} S^{1/2}$$
$$= (by + ty^2) \cdot \left(\frac{y}{2}\right)^{2/3} \cdot S^{1/2}$$
$$= \left(2y^2\left(\sqrt{1+t^2} - t\right) + ty^2\right) \cdot \left(\frac{y}{2}\right)^{2/3} \cdot S^{1/2}$$
$$10n = \left(2y^2\left(\sqrt{10} - 3\right) + 3y^2\right) \cdot \left(\frac{y}{2}\right)^{2/3} \cdot (0.0002)^{\frac{1}{2}}$$
$$10n = (0.324y^2 + 3y^2)\ y^{2/3} \cdot 0.009$$
$$10n = 0.03\ y^{4/3}$$
$$333.33n = y^{4/3}$$
$$y = \sqrt[4]{(333.33 \times 0.012)^3}$$
$$y = 2.83\ m$$

$$b = 2y\left(\sqrt{1+t^2} - t\right)$$
$$= 2(2.83)\left(\sqrt{1+3^2} - 3\right)$$
$$= 0.92\ m$$

$\therefore$ The most economic dimensions are.

$y = 2.83\ m$

$b = 0.92\ m$